Radiation from relativistic collapsing compact objects

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Introduction

• From the Einstein’s gravity, light propagating in the vicinity of astrophysical compact objects is affected by the gravitational field.

• The general relativistic effects might be important for understanding the features of the radiation coming from the NS-like objects.

• The gravitational redshift and bending of light rays emitted by a compact object affect the form and spectrum of the observed signals.
Light Emission from a Collapsing Surface

- First, we consider a sufficiently slowly rotating object.
  - Under this consideration, we can approximate that the background metric is not rotating metric but the Schwarzschild metric for light propagation.

- Second, the radiation emitted by the polar cap, which has size of half angle $\alpha$ of a collapsing surface.
  - We assume that the axis of polar cap is aligned to the direction of a distant observer.

- Third, the radiation emitted during a finite time interval

  With these assumptions, we calculate light curves and the spectrum of this radiation as seen by a distant observer.
What is Polar Cap?

- Polar cap emission region
- Emission points for the same arrival time
- The last tangential ray for each R
- Trajectory of emitted ray

- $R_i$: Initial radius of emission
- $R_f$: Final radius of emission
- $\alpha$: Half-angle of polar cap

- a: The point of the first light with $l=0$ from $R_i$
- b: The point of the first light with $l=0$ from $R_f$
- c: The light is emitted tangentially from $R_i$
- d: The light is emitted tangentially from $R_f$
- e: The point of the ray with $\alpha$ and $R_i$
- f: The point of the ray with $\alpha$ and $R_f$
Basics of Light Propagation

- $\tau$: the proper time as measured by an observer comoving with the collapsing surface.
- In the coordinates $(t, r, \theta, \phi)$, the four-velocity of the collapsing surface is
  \[ v^\mu = \left( \frac{dt}{d\tau}, \frac{dR}{d\tau}, 0, 0 \right). \]
- Define $v_I$ as the invariant radial velocity which measures the proper length change as measured by the proper time of the observer at rest at a given radius.
- Assuming a freely-falling surface with the initial radius $R_0$, the invariant velocity as a function of $R$ is
  \[ v_I = -\sqrt{\frac{2M}{R}} \frac{\sqrt{1 - R/R_0}}{\sqrt{1 - 2M/R_0}}. \]
• The 4-momentum of a photon $p^\mu = (p^t, p^r, p^\theta, 0)$
• Use the impact parameter defined by $l = L/E$

\[ E = -p_t \quad : \text{the energy of photon at infinity} \]
\[ L = p_\theta \quad : \text{the angular momentum of photon} \]

\[ p^r = \sigma EZ, \quad \text{Where} \quad Z(l, r) = \sqrt{1 - \frac{l^2 f(r)}{r^2}}, \quad f = f(r) = 1 - 2M/r \]

( $\sigma$: “+” forward motion of photon or “-” backward motion of photon )
• The possible range of the impact parameter for a photon’s propagation
  – Upper limit: \( l_{max} = R / \sqrt{f(R)} \rightarrow Z(l_{max}, R) = 0 \)
  
  – Especially, for the backward ray, the lower limit:
  \[
  l_T = R / \sqrt{f(R_0)} \rightarrow Z(l_T, R) = -v_I
  \]

\[
0 \leq l \leq l_{max}, \quad l_T \leq l \leq l_{max}
\]

• In this work we consider only \( R > 3\sqrt{3}\sqrt{1 - 2M/R_0 M} \) for the possible region of photon’s propagation.
Bending Angle

- Forward emission: \( \theta_+ = \Theta(l, R) \equiv l \int_R^\infty \frac{dr}{r^2 Z(l, r)} \quad (0 \leq l \leq l_{max}) \)

- Backward emission: \( \theta_- (l, R) = 2\Theta(l, r_t) - \Theta(l, R) \quad (l_T \leq l \leq l_{max}) \)

Dimensionless quantities: \( x = M/r, \quad q \equiv M/R, \quad \hat{l} = l/M \)

\[
\Theta(\hat{l}, q) = \hat{l} \int_0^q \frac{dx}{\sqrt{1 - \hat{l}^2 x^2(1 - 2x)}}
\]
Redshift Factor

\[ \Phi \equiv \nu^{(e)}/\nu^{(o)} \]

\[ \begin{align*}
\nu^{(e)} &: \text{measured by a comoving observer} \\
\nu^{(o)} &: \text{measured by a distant observer}
\end{align*} \]

- For a given ray with the \((l, R)\)

\[ \Phi_{\sigma}(l, R) = \frac{1 - \sigma v_l Z(l, R)}{\sqrt{f} \sqrt{1 - v_l^2}} \]

- For the tangential (or last) ray

\[ \Phi_T = \frac{1}{\sqrt{1 - 2M/R_0}} \]
Arrival Time Difference

- Forward

\[ \Delta t_+(l; \tau, \tau_i) = t^{(e)}(\tau) - t^{(e)}(\tau_i) + T(l, R(\tau)) + R_i - R(\tau) + 2M \ln \frac{R_i - 2M}{R(\tau) - 2M} \]

\[ T(l, R) = \int_R^\infty \frac{dr}{f(r)} \left[ \frac{1}{Z(l, r)} - 1 \right] \]

\[ T(\hat{l}, q) = \hat{l}^2 \int_0^q \frac{1 + \sqrt{1 - \hat{l}^2 x^2 (1 - 2x)}}{\sqrt{1 - \hat{l}^2 x^2 (1 - 2x)}} \, dx \]

- Backward

\[ \Delta t_-(l; \tau, \tau_i) = t^{(e)}(\tau) - t^{(e)}(\tau_i) + 2T(l, r_t) - T(l, R(\tau)) + R_i + R(\tau) - 2r_t + 2M \ln \frac{(R(\tau) - 2M)(R_i - 2M)}{(r_t - 2M)^2} . \]
Flux

\[ F_{\nu}^{(o)}(t) = \frac{2\pi}{r_0^2} \int d\tau l \left| \frac{dl}{d\tau} \right| \Phi^{-4} I_{\nu}^{(e)}(l, \tau) \]

where, \( I_{\nu}^{(e)}(l, \tau) = f(\tau) I^{(e)}(l) \)

-> The isotropic emission : \( I^{(e)}(l) = I^{(e)} \)

\[ F_{\nu}^{(o)}(t) = \frac{2\pi I^{(e)}}{r_0^2} \int d\tau f(\tau) l \left| \frac{dl}{d\tau} \right| \Phi^{-4} \]
Results - Flux

1) \( R_0 = 9M, \ R_i = 6.0M, \ R_f = 4.6M \)

\[ (\delta \equiv \Delta t_\pm / \Delta T, \ \text{where} \ \Delta T \equiv \Delta t_-(l_T, \ \tau_f, \ \tau_i) ) \]
2) \( R_0 = 9M, \ R_i = 6.0M, \ R_f = 4.6M \)
3) $R_0 = 9M$, $R_i = 6.0M$, $R_f = 4.6M$

Polar cap radiation ($\alpha = 0, 5, 10, 15$)  
Polar cap radiation ($\alpha = 20, 40, 60, 80$)
Results – Redshift Factor

1) \( R_0 = 9M, \ R_i = 6.0M, \ R_f = 4.6M \)

Full surface radiation  
Polar cap radiation (\( \alpha = 40 \))
2) $R_0 = 9M, \ R_i = 6.0M, \ R_f = 4.6M$
3) \( R_0 = 9M \), \( R_i = 6.0M \), \( R_f = 4.6M \)

Polar cap radiation (\( \alpha = 10 \))

Polar cap radiation (\( \alpha = 80 \))
Remarks

• In our model, the physical parameters characterizing the collapsing object are its mass $M$ and three of dimensionless parameters $R_0/M$, $R_i/M$, and $R_f/M$.
• In case we don’t know the frequency of the emitted radiation, we can’t determine the redshift factor directly by observing the spectrum of the radiation.
• Nevertheless, if it is possible to determine the frequency $\nu^{(o)}_{last}$ of the last ray with sufficient accuracy, then the relative (normalized) redshift factor $\tilde{\Phi} \equiv \Phi/\Phi_T = \nu^{(o)}/\nu^{(o)}_{last}$ can be determined.
• Then by measuring $\delta_f$ and $\delta_T$ as well as $\tilde{\Phi}_0$ and $\tilde{\Phi}_{max}$ one can determine $R_0/M$, $R_i/M$, and $R_f/M$.
• Once we know $R_0/M$, $\Phi_T$ can be evaluated and we can determine the original frequency of emission as given by $\nu = \Phi_T \nu^{(o)}_{last}$.
• The mass of the collapsing object can be inferred from the observed value of $\Delta t$ to complete the determination of the physical characteristics of the collapsing object.

• From the determined parameters $R_0/M$, $R_i/M$, $R_f/M$, impact parameters, and arrival time, we can estimate the size of polar cap.
  • e.g. When $\alpha = 40$ deg., the bending angle covers 41 deg. $\sim$ 138 deg. for the zero flux between $\delta = 0.2$~$0.3$. Also, after the last backward radiation ($\delta = 0.6$~$1$), the bending angle covers 223 deg. $\sim$ 318 deg. (Emission region: 320 deg. $\sim$ 40 deg. & 140 deg. $\sim$ 220 deg.)
Summary

• We discuss the light curves and the spectral broadening of the radiation emitted from polar cap region of a collapsing object.
• We demonstrate that the spectral broadening change occurs when the radiation from the polar cap of collapsing surface takes place during a finite duration of time.
• The existence of polar cap gives a different shape of flux and redshift factor, which covers only a part of the original curves for the whole surface emission case.
• It is because of the variance of the frequency shifts of the light rays subjected to the gravitational redshift and the Doppler shift of the collapsing surface.